A Method of Determining the Relative Contributions of Individual Equations in Hess' Law Problems

Idea initiated by Haig Mastikian and extended by Prof. John Milligan

This method is similar to the mathematical method of balancing chemical equations. However, here we have a system of equations that is over specified so we can calculate the exact numbers.

Example:

Use Hess' Law to determine the enthalpy of the reaction

C_3H_4Þ + 2 H_2Þ $\rightarrow$ C_3H_8Þ

given

H_2Þ + ½ O_2Þ $\rightarrow$ H_2Oà	ΔH_1
C_3H_4Þ + 4 O_2Þ $\rightarrow$ 3 CO_2Þ + 2 H_2Oà	ΔH_2
C_3H_8Þ + 5 O_2Þ $\rightarrow$ 3 CO_2Þ + 4 H_2Oà	ΔH_3

First we will multiply each equation by a generic constant (i.e. a, b, c, etc.)

a(H_2Þ + ½ O_2Þ $\rightarrow$ H_2Oà)	ΔH_1
b(C_3H_4Þ + 4 O_2Þ $\rightarrow$ 3 CO_2Þ + 2 H_2Oà)	ΔH_2
c(C_3H_8Þ + 5 O_2Þ $\rightarrow$ 3 CO_2Þ + 4 H_2Oà)	ΔH_3

We then set up an equation for each ***compound*** in the given reactions. The coefficients of reactants are positive numbers; those of products are negative numbers. The equations are equal to the coefficient in the overall equation. If the substance is not present in the overall equation, the equation equals zero.

H_2	$a = 2$
O_2	$\frac{1}{2} a + 4 b + 5 c = 0$
H_2O	$-a + (-2b) + (-4c) = 0$
C_3H_4	$b = 1$

CO_2 -3b + -3c = 0
C_3H_8 c = -1

In this example the solution is given explicitly a = 2, b = 1 and c = -1.

Let’s look at a more complicated example:

Given

C(graphite) + ½ O_2Þ → COÞ	ΔH = -110.5 kJ
COÞ + ½ O_2Þ → CO_2Þ	ΔH = -283.0 kJ
H_2Þ + ½ O_2Þ → H_2Oà	ΔH = -285.8 kJ
C(graphite) + 2 H_2Þ → CH_4Þ	ΔH = -74.81 kJ
CH_4Þ + 2 O_2Þ → CO_2Þ + 2 H_2Oà	ΔH = -890.3 kJ

Find ΔH for

4 COÞ + 8 H_2Þ → 3 CH_4Þ + CO_2Þ + 2 H_2Oà

Set up equations

a(C(graphite) + ½ O_2Þ → COÞ	ΔH = -110.5 kJ)
b(COÞ + ½ O_2Þ → CO_2Þ	ΔH = -283.0 kJ)
c(H_2Þ + ½ O_2Þ → H_2Oà	ΔH = -285.8 kJ)
d(C(graphite) + 2 H_2Þ → CH_4Þ	ΔH = -74.81 kJ)
e(CH_4Þ + 2 O_2Þ → CO_2Þ + 2 H_2Oà	ΔH = -890.3 kJ)

C a + d = 0
O_2 ½ a + ½ b + ½ c + 2 e = 0
CO -a + b = 4
CO_2 -b – e = -1
H_2 c + 2d = 8
CH_4 -d + e = -3
H_2O -c + -2e = -2

a = -d c = 8 – 2d
b = 4 + a = 4 – d e = – 3 + d

½ (–d) + ½ (4 – d) + ½ (8 – 2d) + 2 (–3 + d) = 0
–½ d + 2 – ½ d + 4 – d – 6 + 2 d = 0

All the d's and the numbers cancel to zero so d can be anything.

$$0d + 0 = 0$$

Here, however, it must be zero because just any number will not make sense. Because d = 0 then c = 8 and e = –3 and a = 0 and b = 4. Let's check:

0(C(graphite) + ½ O_2Þ → COÞ	ΔH = -110.5 kJ)
4(COÞ + ½ O_2Þ → CO_2Þ	ΔH = -283.0 kJ)
8(H_2Þ + ½ O_2Þ → H_2Oà	ΔH = -285.8 kJ)
0(C(graphite) + 2 H_2Þ → CH_4Þ	ΔH = -74.81 kJ)
–3(CH_4Þ + 2 O_2Þ → CO_2Þ + 2 H_2Oà	ΔH = -890.3 kJ)

Rewriting this gives

4 COÞ + 2 O_2Þ → 4 CO_2Þ	ΔH = -1132.0 kJ
8 H_2Þ + 4 O_2Þ → 8 H_2Oà	ΔH = -2286.4 kJ
3 CO_2Þ + 6 H_2Oà → 3 CH_4Þ + 6 O_2Þ	ΔH = +2670.9 kJ

4 COÞ + 8 H_2Þ → CO_2Þ + 3 CH_4Þ + 2 H_2Oà

This is what we are trying to find so the enthalpy of this reaction is:

$$\Delta H = -1132.0\text{ kJ} + -2286.4\text{ kJ} + 2670.9\text{ kJ} = -747.5\text{ kJ}$$