Number and Measurement

All of science relies on measurements of some sort. Physics measures mass, velocity, and acceleration. Chemistry measures mass, temperature, volume, and pressure. All measurements have two components: a number and a unit. For now, we are going to focus on the first of these, the number.

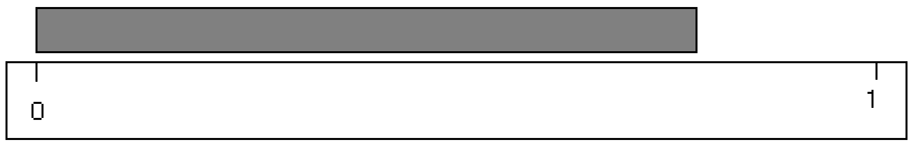
Number tells us how much of something there is. We never have a measurement without a number. I don't say that my height is inches or meters. There must be some amount of these units. That is the number portion of the measurement.

The number part of the measurement is the part that is susceptible to error. We **can** make a mistake in the unit we use. This can also happen with the number. If we make this type of mistake, misreading the number or an error arising from a malfunction in the measuring device, it is called a ***systematic error*** and is always in the same direction for that measurement. This kind of error can be found and eliminated by re-measuring or repairing or recalibrating the device.

There is another kind of error. This error arises because no measuring instrument is 100% precise. This is called ***random error***. It is random because it can make the measurement too high or too low. Each of these types of error affects our measurement.

We can qualify a measurement by specifying one of two numbers: the ***accuracy*** and ***precision***. Although these terms are usually used interchangeably, they have distinct meanings. Accuracy refers to how close to the correct value a measurement is. Precision refers to how close individual measurements are to each other. Usually in science, we take multiple measurements of a quantity. These individual measurements are then averaged together. The reason for this is twofold. First, multiple measurements reduces the possibility of making a mistake. It reduces the systematic error. Second, it also makes the measurement more likely to be close to the correct value. Averaging reduces the random error.

Because of random error, all measurements carry a degree of uncertainty. This is measured by the number of digits in the measurement. If we have a meter stick that has only two markings on it, at 0 m and at 1 m, we can make only an approximation to the correct value.



In the picture above, we can say, at best, that the bar is about 0.8 m long. If our meter stick had more markings, our answer will be more accurate. Whenever we make a measurement, we always estimate the last digit. In this example, the “0” is known. We know that we have at least zero meters and not quite 1 meter. The “.8” is an estimate. All measurements contain all digits that are certain plus one that is an estimate. When we count all of those digits, it tells us how many ***significant figures*** there are in the measurement. Numbers should never be reported with more significant figures than are warranted by our measuring device. Another way of thinking about this is that we always estimate the answer to $1/10^{th}$ of the smallest division that is marked. A meter stick that is marked every millimeter should be used to estimate a length to the nearest $1/10^{th}$ of a millimeter (±0.0001 m).

When we are making the measurements, we know how many “sig figs” there should be. What if we are looking at someone else’s measurement? How many sig figs should there be? The distance to the Moon is 384,000 km. Is this accurate to the nearest kilometer or the nearest thousand kilometers? There is a more precise way of stating this kind of number. We will look at this soon.

How do we determine the number of sig figs in a measurement? We have rules that tell us this.

1. All non-zero digits are significant.
2. Leading zeroes are never significant.
3. Trailing zeroes are not significant unless the number is in scientific notation.
4. Zeroes between non-zero digits are significant.

Scientific notation will be discussed later.

How do mathematical operations affect the number of significant figures in a number? We have rules for this too.

When we are adding (or subtracting) two numbers, the answer cannot have more decimal places than the number with the fewest number of decimal places.

Example:

124.3	1 decimal place
+ 54.	0 decimal places
____	
178.3	According to the calculator
178	With the correct number of decimal places.

When we are multiplying (or dividing) two numbers, the answer cannot have more sig figs than the number with the fewest number of sig figs.

Example:

124.3	4 sig figs
$\times$54.	2 sig figs
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6712.2	According to the calculator
6.7×10^3	With the correct number of sig figs.

Scientific Notation

Scientific Notation is a method of representing very large or very small numbers. This is done using powers of ten (10^x). Earlier we saw the distance to the moon was 384,000 km. This is an ambiguous number. We can represent this number more accurately (i.e., with the correct number of sig figs) by writing it in scientific notation. This number then becomes 3.84×10^5 km. The exponent on the 10 indicates the number of places to the left we have to move the decimal point. If it is a negative exponent, the decimal had to be moved to the right (or the original number was less than one). The number is written with only the necessary number of significant figures.

Mathematical operations

When doing operations with numbers in scientific notation, the same rules for sig figs apply.

Addition/Subtraction

The numbers should first be written with the same exponent. Then the same rules as before apply.

Multiplication/Division

The main parts of the numbers are multiplied and the exponents are added. The same rules as before apply for sig figs.