

Units and Dimensional Analysis

Metric System of Units

The system of units most used in the sciences is the metric system. This is a system based on powers of 10. The English (or American) system of units is based on various powers of 2 (i.e., 2 cups is a pint, 4 quarts is a gallon, etc.) or other obscure factors (i.e., 12 inches is a foot or 2000 pounds is a ton). In order to change from one unit to another in the metric system is a matter of multiplying or dividing by powers of 10. We can also specify various units by the use of prefixes. These prefixes and their meanings must be memorized. The main prefixes you should know are:

Prefix	Symbol	Power of 10
Pico	p	10^{-12}
Nano	n	10^{-9}
Micro	μ	10^{-6}
Milli	m	10^{-3}
Centi	c	10^{-2}
Kilo	k	10^3
Mega	M	10^6

Once you know these prefixes, you can convert from one unit in the metric system to another.

Mass units

It is important to understand the concept of mass. Mass is a measure of the amount of matter that an object contains. This is not to be confused with weight, which is a measure of the gravitational pull of the Earth on a mass. The unit of mass on the metric system is the **gram (g)**. This is a relatively small amount of mass. One gulp of water has a mass of approximately 20 grams. There is a related system of units called **SI** (for System Internationale). In SI, the unit of mass is the kilogram (10^3 grams). In SI, the mass unit is defined as the mass of a platinum-iridium cylinder held in Paris, France.

Length units

In the metric system and SI, the unit of length is the **meter (m)**. The definition of length was originally the one ten-thousandth of the distance from the North Pole to the Equator through Paris, France. It is currently defined as a given number of wavelengths of the emitted light from a Cesium atom. This is a more accurate and reproducible way of defining a unit of length.

Volume units

In the metric system, the unit of volume is a derived unit. It is derived from the fact that volume can be defined as length raised to third power. The unit for volume in the metric system is the cubic meter. In SI the unit is the ***liter (L)***.

Time units

In both the metric system and SI the unit of time is the ***second (s)***.

Temperature units

The metric unit of temperature is the ***degree Celsius (°C)***. In SI the units is the ***Kelvin (K)***. Note that in Kelvin there is no degree sign. The degree size for both is the same. The difference in these units is where the zero point is defined. In degrees Celsius, the zero is defined to be the freezing point of water. In Kelvin, it is absolute zero, the coldest temperature possible. As a consequence of the definition of the Kelvin scale, there is no such thing as a negative Kelvin temperature.

Mathematical operations involving units

When using units in mathematical operations, they act just like numbers. When adding two numbers with units, they must have the same units. You cannot add 2.3 m to 54.3°C. When multiplying (or dividing) numbers with units, the units also multiply (or divide). For example, 12.3 m × 14.2 m is 175 m² (m×m). If we divide 2.5 m by 1.3 s we get 1.9 m/s (which can also be written as m s⁻¹). If we divide units that are the same they cancel each other out just like numbers.

Conversion Factors

A conversion factor is a number involving units that allows us to convert from one kind of unit to another. The conversion factor is usually a statement of equivalence. Sometimes it is also a statement of equality. Conversion factors such as density or speed are statements of equivalence. The density of water is approximately 1.0 g mL⁻¹. This means that, for water, 1.0 g of water is equivalent to 1 mL of water. Unit conversions from one unit to another of the same type of unit are statements of equality. For example, 12 inches is equal to 1 foot.

Conversion factors are ratios. They relate one type of unit to another. Because they are ratios they can be used in either direction. We can state the above conversion factor of the density of water as:

$$\frac{1.0 \text{ g}}{1 \text{ mL}}$$

Or we can look at it “upside-down.”

$$\frac{1 \text{ mL}}{1.0 \text{ g}}$$

Both are equally correct.

Dimensional Analysis

Dimensional analysis is a process where conversion factors are used to cancel units so that one unit is converted into another. This process uses the concepts that have been discussed earlier. The basic idea behind dimensional analysis is as follows:

- Identify the unit(s) that is(are) given. This will be a clue as to how you will proceed with the rest of the problem.
- Identify the unit that is requested. This is what you are asked to solve.
- Identify any conversion factors that you may need. These may include English-to-English, English-to-metric, or equivalence conversion factors such as speed or density.
- Arrange the conversion factors such that the unit in the conversion factor cancels out the unit you have. Continue in this manner until you have the desired units.
- Do the math.

How is this applied in practice? Suppose we wish to know how many inches are in 22.4 km. Lets look at this problem with the above steps in mind.

- Identify the unit that is given.

In this problem the given unit is “km.” We are given 22.4 km.

- Identify the unit that is requested.

In this problem we are asked to find this distance in inches. “Inches” is the unit we need.

- Identify any conversion factors.

We will need the following conversion factors:

$$1 \text{ km} = 10^3 \text{ m} \qquad 1 \text{ cm} = 10^{-2} \text{ m} \qquad 1 \text{ in} = 2.54 \text{ cm}$$

- Arrange the conversion factors such that units cancel.

We then have:

$$22.4 \cancel{\text{ km}} \times \frac{10^3 \cancel{\text{ m}}}{1 \cancel{\text{ km}}} \times \frac{1 \cancel{\text{ cm}}}{10^{-2} \cancel{\text{ m}}} \times \frac{1 \text{ in}}{2.54 \cancel{\text{ cm}}}$$

Notice how each conversion factor cancels the unit that came before it, leaving, in the end, the desired units.

- Do the math.

$$22.4 \text{ km} \times \frac{10^3 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ cm}}{10^{-2} \text{ m}} \times \frac{1 \text{ in}}{2.54 \text{ cm}} = 8.82 \times 10^5 \text{ in}$$

Be sure to express the answer with the correct number of significant figures.

Density

Density is a conversion factor. It is a conversion factor that makes a statement of equivalence. Density converts between the mass of an object and its volume. If we know the mass of an object, we can find its volume and vice versa.

Temperature Scales

In chemistry, the main unit of temperature we use is degrees Celsius. We also use the unit Kelvin in some instances. Sometimes, however, the temperature is given in units of degrees Fahrenheit. We need to be able to convert between these units.

Converting between degrees Celsius and Kelvin is relatively easy because the units are the same size. The only difference is where each scale starts. The Kelvin scale starts at absolute zero (0 K or -273.15°C). The Celsius scale has as its zero point the freezing point of water (0 $^\circ\text{C}$ or +273.15 K). We can use these differences to establish a conversion between the two units. In order to convert from degrees Celsius to Kelvin we need to add 273.15 $^\circ\text{C}$ and then multiply by 1 K/1 $^\circ\text{C}$. To go the other way we subtract 273.15 K and then multiply by 1 $^\circ\text{C}/1 \text{ K}$. So we have the following conversion factors:

$$K = (^\circ\text{C} + 273.15^\circ\text{C}) \times \frac{1\text{K}}{1^\circ\text{C}}$$

$$^\circ\text{C} = (K - 273.15\text{K}) \times \frac{1^\circ\text{C}}{1\text{K}}$$

Converting between Fahrenheit and Celsius is very much the same. Here we need to remember that for every 9°F there are 5°C. In addition to the different unit size we also have different zero points. In the Fahrenheit scale the zero is defined as the temperature of a mixture that contains equal masses of ice and salt (0°F, -17.78°C). The freezing point of water on the Fahrenheit scale is 32 °F. This gives us the following conversion factors:

$$^{\circ}\text{C} = (^{\circ}\text{F} - 32^{\circ}\text{C}) \times \frac{5^{\circ}\text{C}}{9^{\circ}\text{F}}$$
$$^{\circ}\text{F} = \left(^{\circ}\text{C} \times \frac{9^{\circ}\text{F}}{5^{\circ}\text{C}} \right) + 32^{\circ}\text{F}$$

Percent units

Percent units are another type of equivalency unit. The unit shows how many parts there are for every 100 parts of the whole. This will allow us to convert between the parts present and the whole that is present.